

## SIR GURUDAS MAHAVIDYALAYA

Internal Examination - 2020

First Year (1+1+1) MTMA, Paper - 1

Time - 2Hr., Full Marks -50

Attempt any five questions.

1. If  $x, y, z$  are positive real numbers and  $x + y + z = 1$ , prove that

$$8xyz \leq (1-x)(1-y)(1-z) \leq \frac{8}{27}.$$

2. If  $\alpha, \beta, \gamma$  are the roots of the equation  $x^3 + px + q = 0$ , find the equation whose roots are  $(\alpha - \beta)^2, (\beta - \gamma)^2, (\gamma - \alpha)^2$ .

3. Solve  $x^3 - 3x + 1 = 0$  by Cardan's method.

4. Let  $f : A \rightarrow B$  and  $g : B \rightarrow C$  be two mappings such that  $gof$  is injective and  $f$  is surjective. Prove that  $g$  is injective.

5. Let  $(G, o)$  be a semigroup and for any two elements  $a$  and  $b$  in  $G$ ,  $aox = b$  and  $yoa = b$  has a solution in  $G$ . Then prove that  $(G, o)$  is a group.

6. Show that the director circle of the conic  $\frac{l}{r} = 1 + e \cos \theta$  is

$$r^2(1 - e^2) + 2ler \cos \theta - 2l^2 = 0.$$

7. Prove by vector method in triangle ABC,  $C = a \cos B + b \cos A$ , where the symbols have their usual meanings.