Send answer script to the following email ID:
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B.SC. PART-I EXAMINATION-2020
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Physics Honours
Subject: PHSA
Paper –I

Time: 2Hrs **Full Marks: 50**

ANSWERS ANY FIVE QUESTIONS

1. ANSWER ANY **FIVE** OF THE FOLLOWING QUESTIONS 2x5=10

(a) Define polar and axial vectors. Give one example of each.

b) If $\vec{A}$ and $\vec{B}$ are each irrotational, prove that $\vec{A}\times\vec{B}$ is solenoidal.

c) Prove that

$$\vec{A}\times(\vec{B}\times\vec{C})+\vec{B}\times(\vec{C}\times\vec{A})+\vec{C}\times(\vec{A}\times\vec{B})=0$$

d) Given two vectors $\vec{A}=a_1\hat{i}+a_2\hat{j}+a_3\hat{k}$ and $\vec{B}=b_1\hat{i}+b_2\hat{j}+b_3\hat{k}$. Find the condition that the two vectors are parallel.

e) When a differential equation is called linear?

f) Find the order and degree of the differential equation

$$\frac{\left[1+\left(\frac{dy}{dx}\right)\right]^{\frac{4}{3}}}{\frac{d^2y}{dx^2}}=C, \quad C = \text{Constant.}$$

g) A particle moves so that its position vector is $\vec{r}=\cos\omega t\,\hat{i}+\sin\omega t\,\hat{j}$, where ω is a constant. Show that the velocity of the particle is perpendicular to its position vector.

2. Answer any **five** questions : 2×5=10

a) Find the volume of the parallelopiped with sides $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$, $\vec{b} = 4\hat{i} + 5\hat{j} + 6\hat{k}$ & $\vec{c} = 7\hat{i} + 8\hat{j} + 10\hat{k}$.

b) Find the unit vector normal to the surface of the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ at the point $\left(\frac{a}{\sqrt{3}}, \frac{b}{\sqrt{3}}, \frac{c}{\sqrt{3}}\right)$.

c) Prove the following $\vec{\nabla} r^n = n r^{n-2} \vec{r}$.

d) Evaluate $\int \vec{A} \times \frac{d^2 \vec{A}}{dt^2}\, dt$.

e) Express the velocity $\vec{v}$ in cylindrical coordinate.

f) Solve $e^x \sin y\, dx + (e^x + 1)\cos y\, dy = 0$.

3. Answer any **two** questions : 5×2=10

a) Verify Stoke's theorem for the vector $\vec{A} = (x^2 - y^2)\hat{i} + 2xy\hat{j}$ over the rectangle in the (x,y) plane bounded by the lines $x = 0$, $x = a$, $y = 0$ & $y = b$. 5

b) Solve $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = xe^x$.

Show that the two functions sin2x, cos2x are independent solutions of $y'' + 4y = 0$. 4+1

c) Find the expression of Laplacian of a function in spherical co-ordinate system. Hence find $\nabla^2\Psi$ for $\Psi(x, y, z) = \frac{xz}{x^2 + y^2 + z^2}$. 3+2

4.

Answer any **two** questions : $5\times2=10$

a) Verify Divergence theorem for $\vec{F}=(x^2-yz)\hat{i}+(y^2-zx)\hat{j}+(z^2-xy)\hat{k}$ taken over the rectangular parallelopiped $0\le x\le a, 0\le y\le b, 0\le z\le c$.

b) Solve $\frac{dy}{dx}=(4x+y+1)^2$ if $y(0)=1$.

c) Evaluate the expression of divergence $(\vec{\nabla}.)$ and curl $(\vec{\nabla}\times)$ in general orthogonal curvilinear coordinate system. $2\frac{1}{2}+2\frac{1}{2}$

5.

Answer any **five** questions : $2\times5=10$

a) Find the directional derivative of $F=2xy+z^2$ at point (1, −1, 3) in the direction of $\hat{i}+2\hat{j}+3\hat{k}$.

b) Prove that $(\vec{A}\times\vec{B})^2=A^2B^2-(\vec{A}.\vec{B})^2$.

c) State Green's theorem of vector calculus.

d) If linear velocity $\vec{v}=\vec{\omega}\times\vec{r}$, prove that $\vec{\omega}=\frac{1}{2}\vec{\nabla}\times\vec{v}$, where $\vec{\omega}$ is a constant vector (angular velocity). Interpret the result.

e) Find the elementary volume element in spherical polar coordinate system.

f) Find the Wronskian of the solutions of the differential equation $y''+4y=0$.

g) Find the Particular Integral of $\frac{d^3y}{dx^3}+y=\cos(2x-1)$.

6. Answer any five questions : $[2\times5=10]$

a) Write down the Taylor's series expansion of a function f(x) about a point x = a.

b) In which physical problems do you apply the method of Lagrange's undetermined multipliers?

c) Find the value of p for which the following three vectors are coplanar:

$$\vec{a} = 3\hat{i} + 2\hat{j} + \hat{k},\ \vec{b} = 3\hat{i} + 4\hat{j} + 5\hat{k},\ \vec{c} = \hat{i} + \hat{j} + p\hat{k}.$$

d) What is the physical significance of gradient of a scalar?

e) Find the order and degree of the following differential equation

$$\left[1+\left(\frac{dy}{dx}\right)^2\right]^3 = c^2\left(\frac{d^2y}{dx^2}\right)^2$$

f) Find the Integrating Factor of the following differential equation:

$$(x+1)\frac{dy}{dx} - y = e^{3x}(x+1)^2.$$

g) Find $\vec{\nabla}\left(\frac{1}{r}\right)$.

h) Find the value of $\int_{-\infty}^{\infty} \delta(x) e^{ikx} dx$.

7.

(a) State Stoke's theorem. Verify it for

$$\vec{A} = (2x - y)\hat{i} - yz^2\hat{j} - y^2z\hat{k}$$

considering the upper half surface of sphere $x^2 + y^2 + z^2 = 1$. 1 + 3

(b) Express $f(x) = x$ as a half-range sine series in the interval $0 < x < 2$. 3

(c) Evaluate

$$\vec{\nabla} \cdot \left\{ r\, \vec{\nabla}\left(\frac{1}{r^3}\right)\right\}.$$

where $r = (x^2 + y^2 + z^2)^{1/2}$ 3